

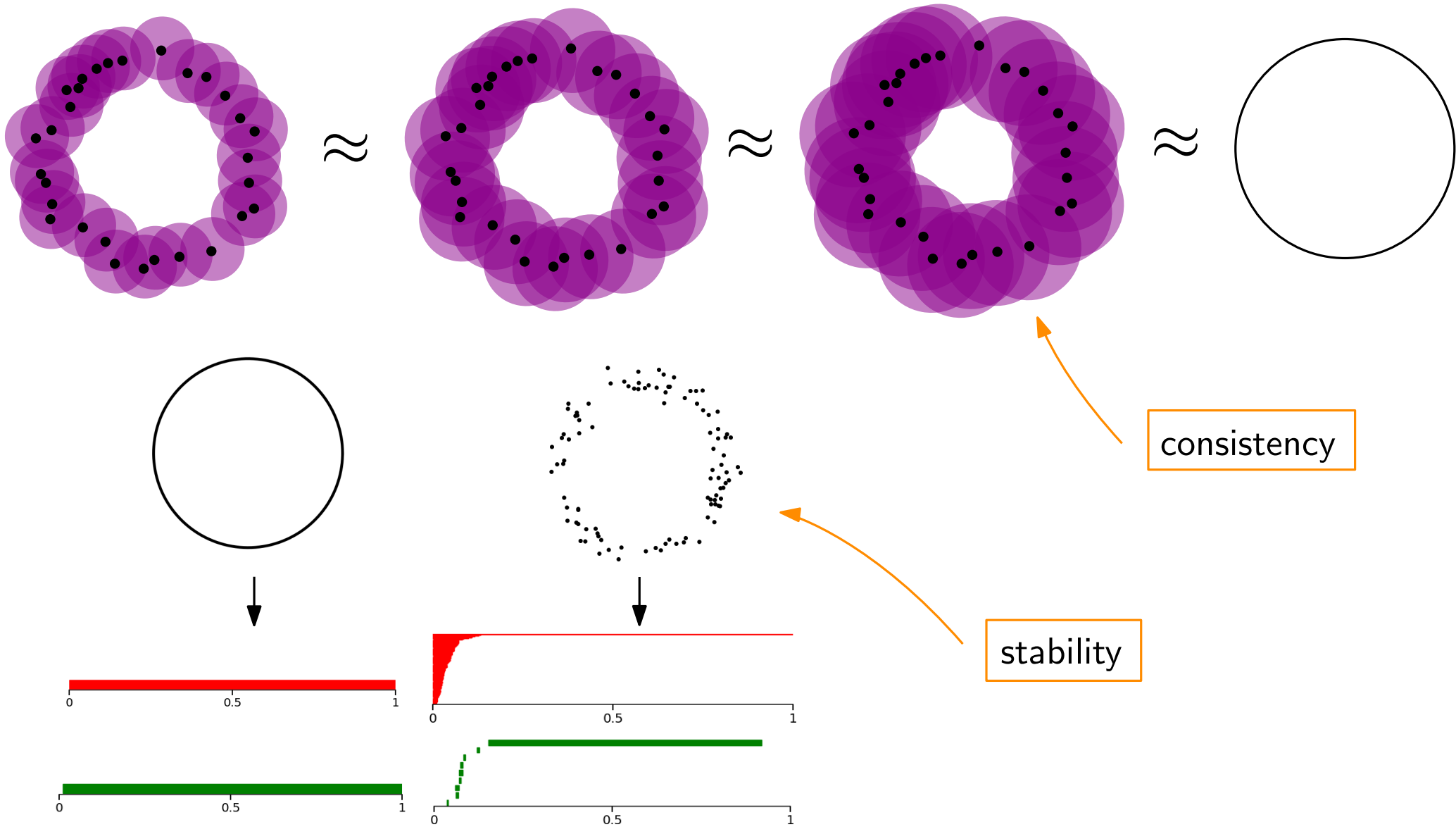
EMAp Summer Course

Topological Data Analysis with Persistent Homology

<https://raphaeltinarrage.github.io/EMAp.html>

Lesson 11: Persistent homology in practice

We have studied the mathematical foundations of persistent homology.



However, its use in practice requires some expertise.

I - Variations on persistent homology

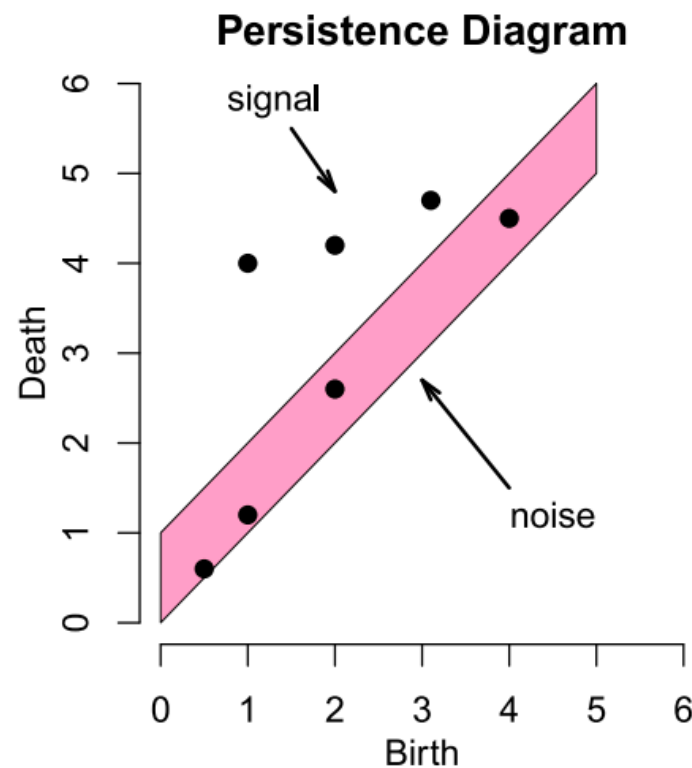
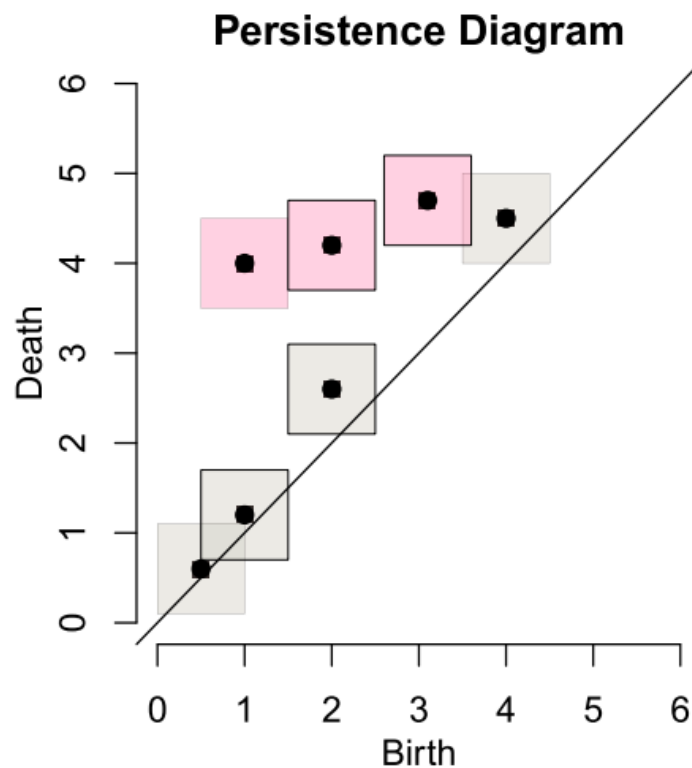
(II - Tutorial)

Statistical aspects of persistent homology 4/14

Brittany Terese Fasy, Fabrizio Lecci, Alessandro Rinaldo, Larry Wasserman, Sivaraman Balakrishnan and Aarti Singh, [Confidence sets for persistence diagrams](#), 2014

<https://arxiv.org/pdf/1303.7117.pdf>

→ Given a barcode, how to determine statistically what is noise and what is not?



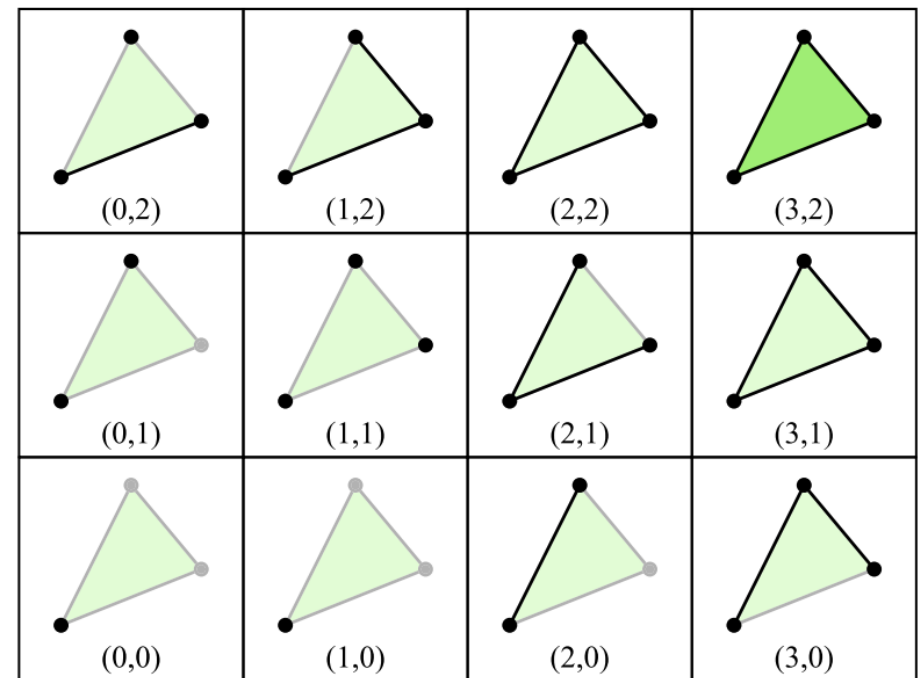
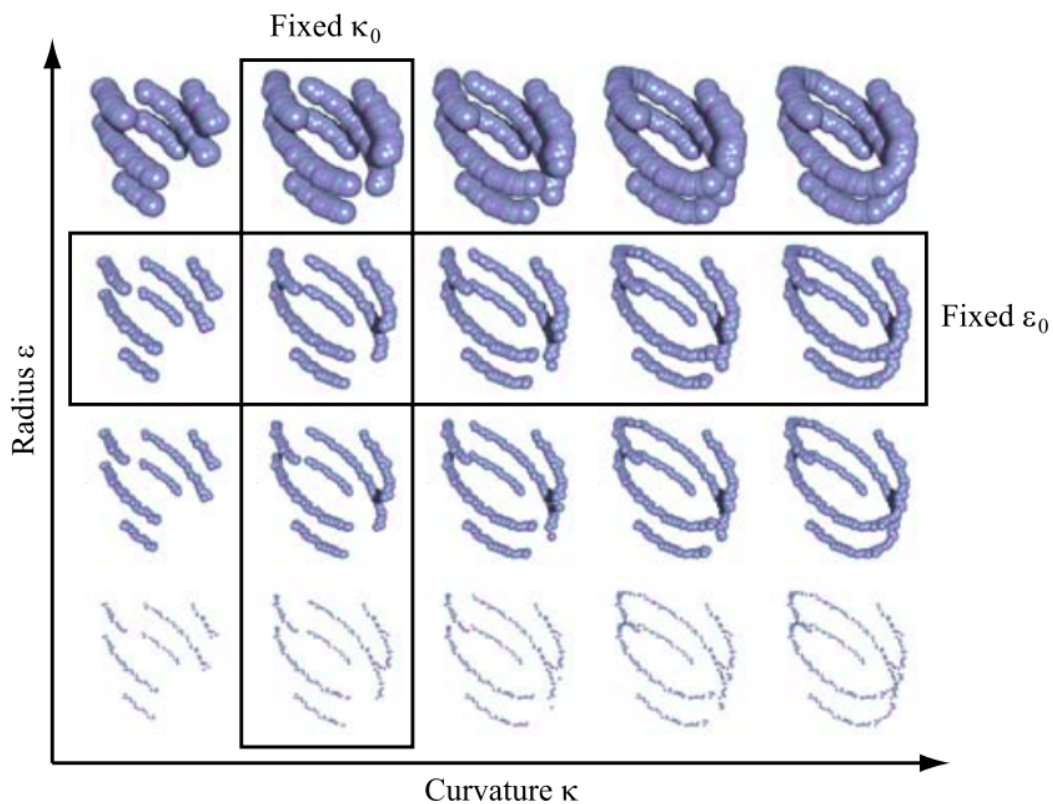
Higher-dimensional persistence

5/14

Gunnar Carlsson, Afra Zomorodian, *The Theory of Multidimensional Persistence*, 2009

<https://link.springer.com/article/10.1007/s00454-009-9176-0>

→ What if our filtration is not indexed only by $t \in \mathbb{R}^+$?



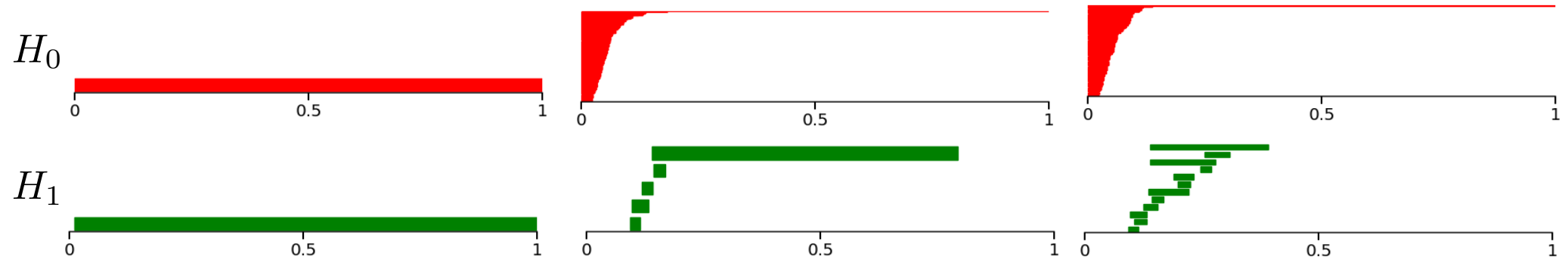
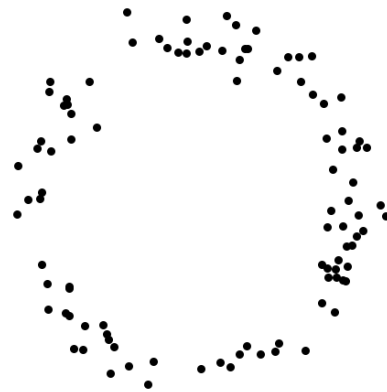
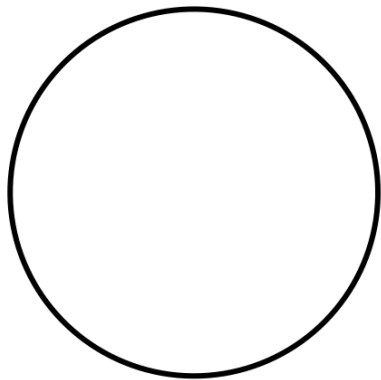
Wasserstein stability

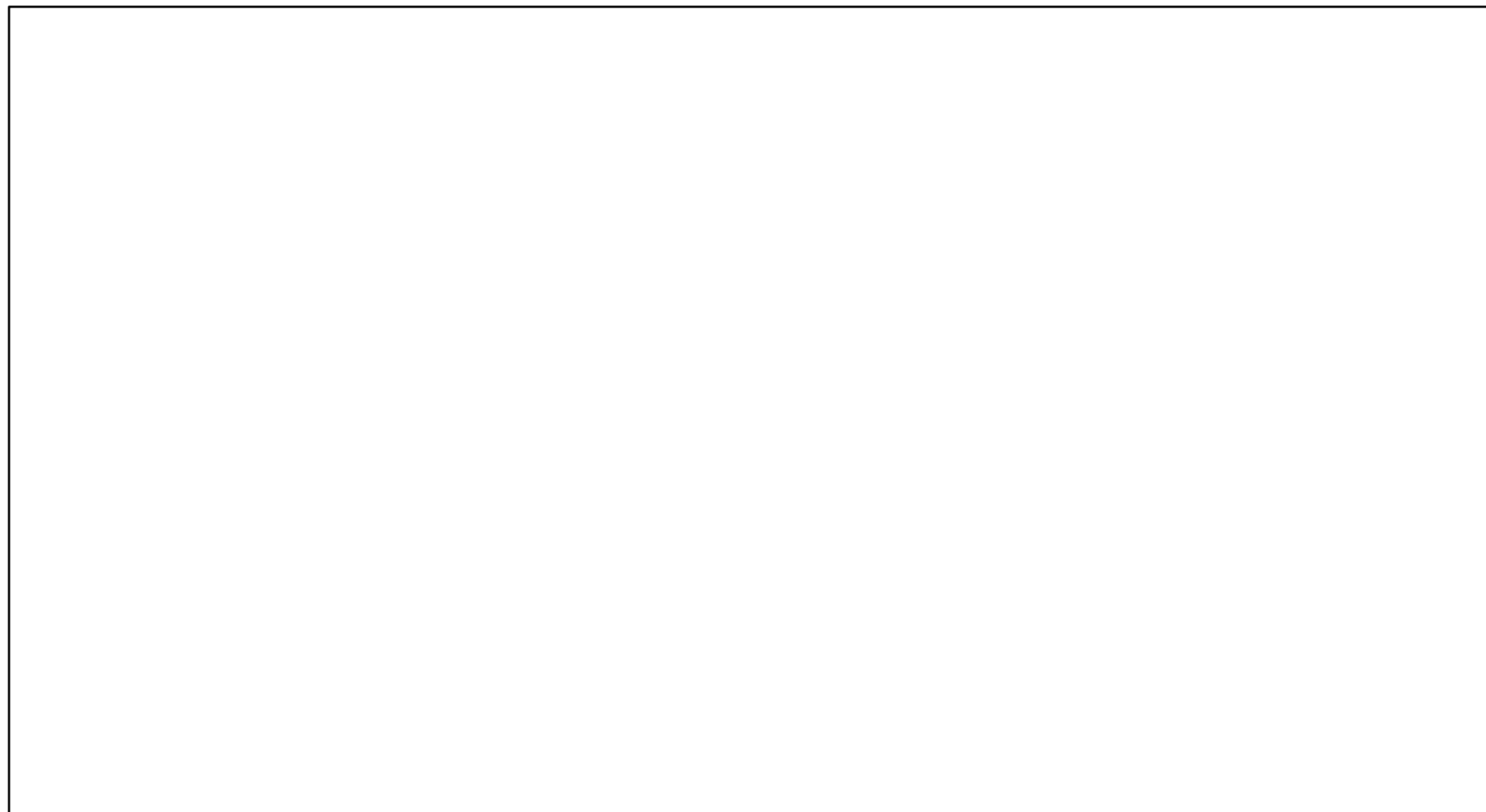
6/14 (1/2)

Hirokazu Anai, Frédéric Chazal, Marc Glisse, Yuichi Ike, Hiroya Inakoshi, Raphaël T., Yuhei Umeda, [DTM-based filtrations](#), 2020

<https://arxiv.org/abs/1811.04757>

→ When our dataset is not close to an underlying object in **Hausdorff distance**





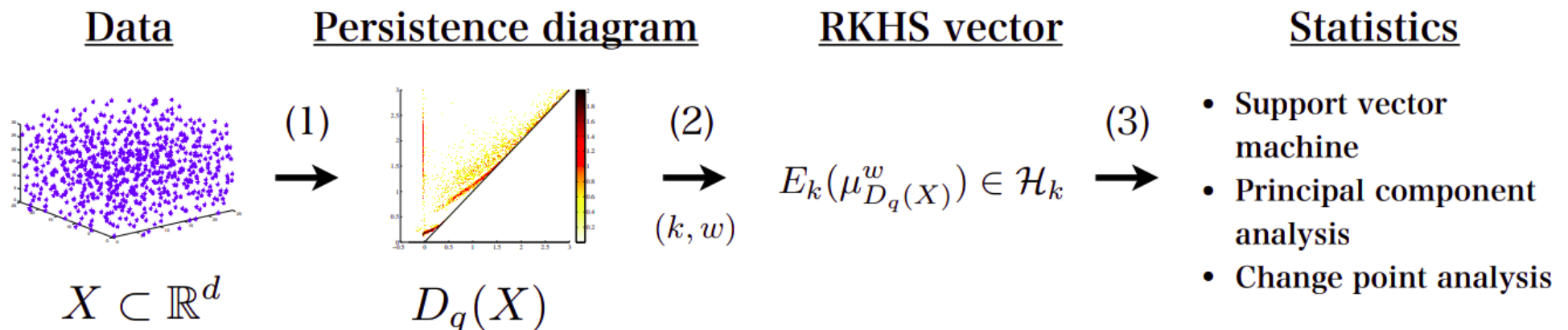
Mathieu Carrière, Marco Cuturi, Steve Oudot, [Sliced Wasserstein Kernel for Persistence Diagrams](#), 2017

<https://arxiv.org/abs/1706.03358>

Genki Kusano, Kenji Fukumizu, Yasuaki Hiraoka, [Kernel Method for Persistence Diagrams via Kernel Embedding and Weight Factor](#), 2018

<https://www.jmlr.org/papers/volume18/17-317/17-317.pdf>

→ Barcodes are not subsets of some Euclidean space, hence usual machine learning methods cannot be used directly



Topological layer in Neural Networks

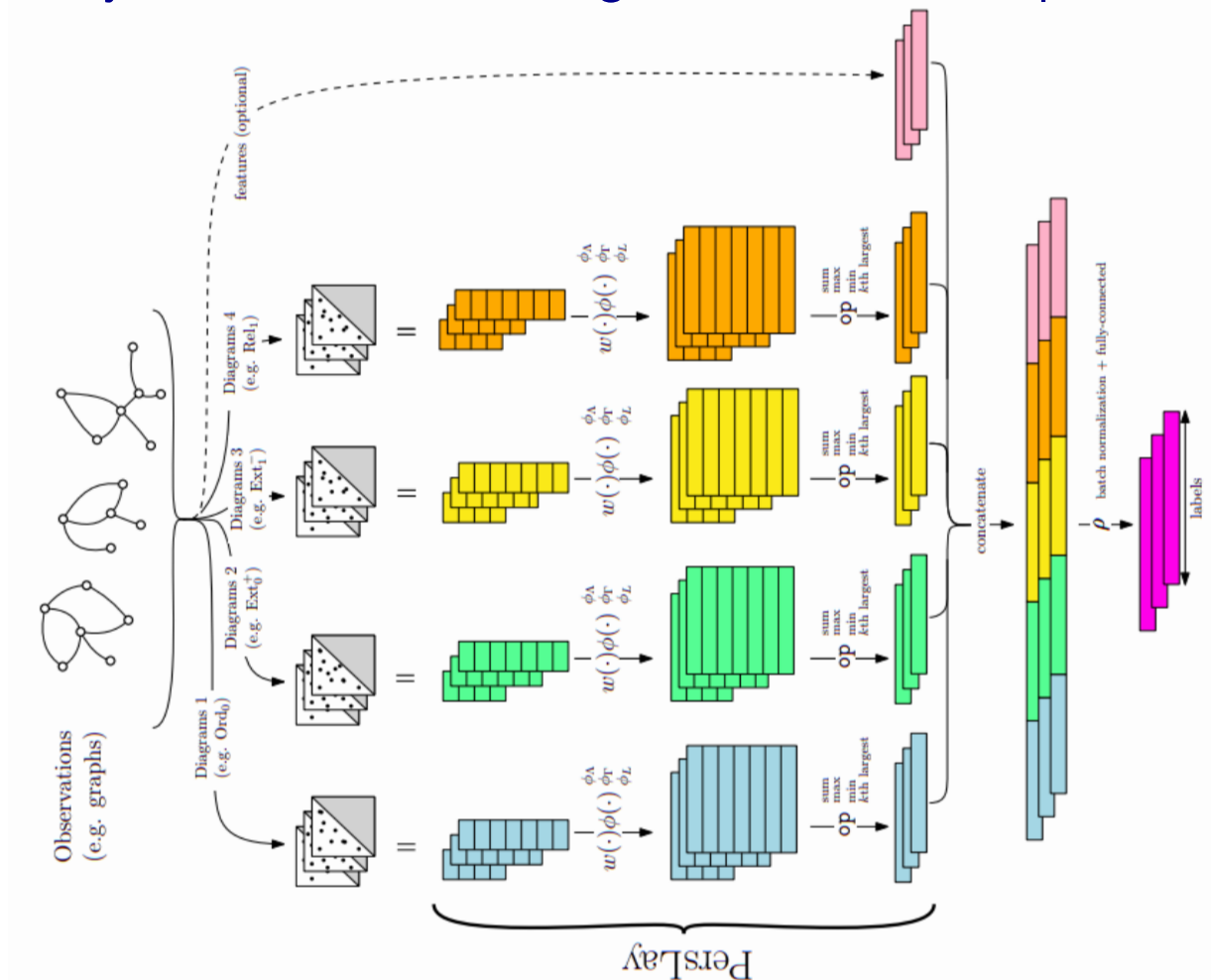
8/14

Rickard Brüel-Gabrielsson, Bradley J. Nelson, Anjan Dwaraknath, Primoz Skraba, Leonidas J. Guibas, Gunnar Carlsson, [A Topology Layer for Machine Learning](#), 2019

<https://arxiv.org/abs/1905.12200>

Mathieu Carrière, Frédéric Chazal, Yuichi Ike, Théo Lacombe, Martin Royer, Yuhei Umeda, [PersLay: A Neural Network Layer for Persistence Diagrams and New Graph Topological Signatures](#), 2019

<https://arxiv.org/abs/1904.09378>



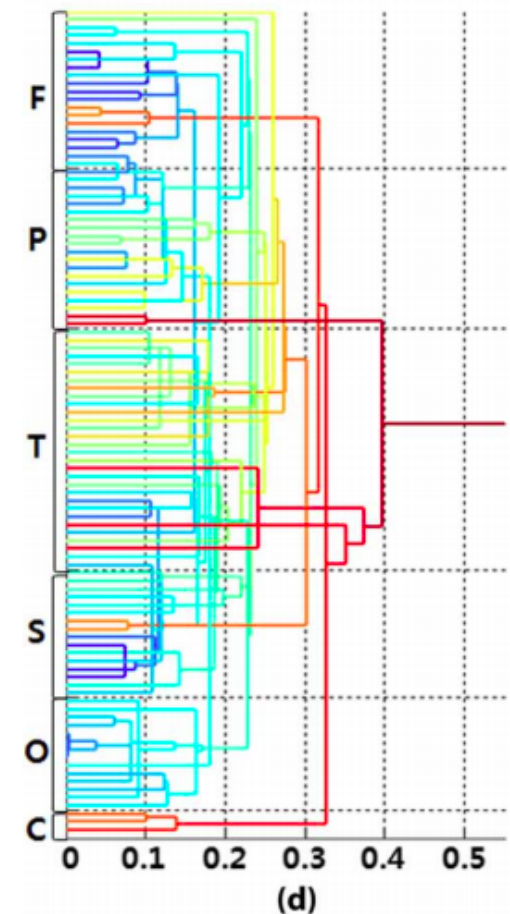
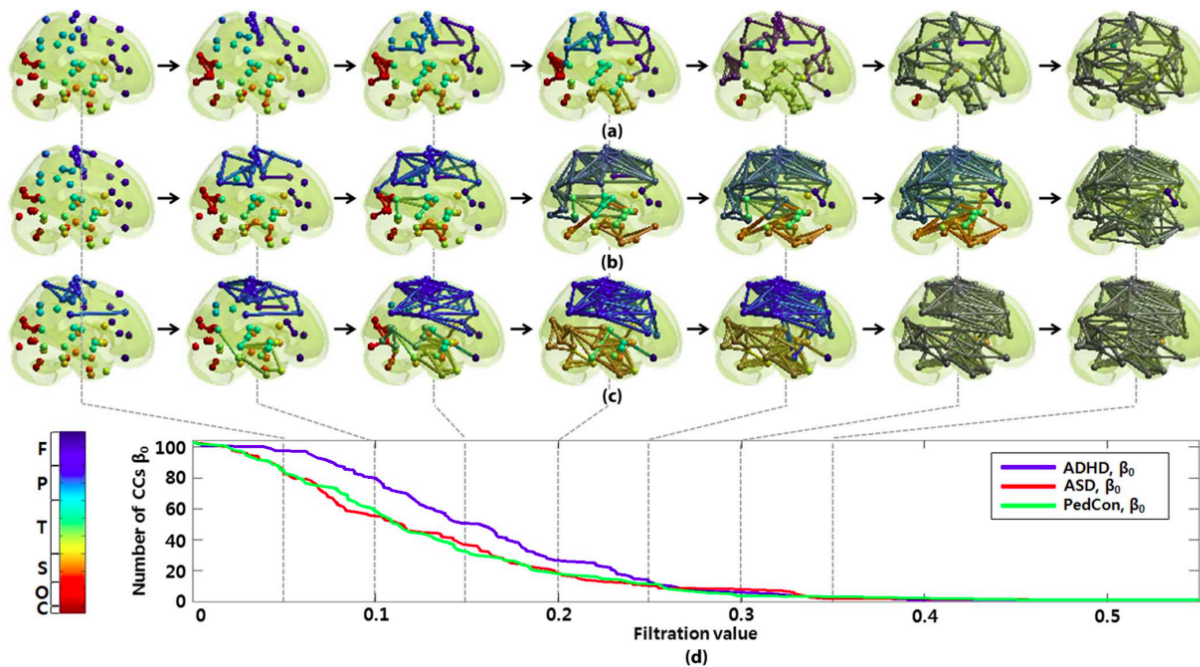
Hierarchical clustering

9/14

Hyekyoung Lee, Hyejin Kang, Moo K Chung, Bung-Nyun Kim, Dong Soo Lee,
Persistent brain network homology from the perspective of dendrogram, 2012

<http://pages.stat.wisc.edu/~mchung/papers/lee.2012.TMI.pdf>

→ H_0 -persistent homology induces a hierarchical clustering



Frédéric Chazal, Steve Oudot, Primoz Skraba, Leonidas J. Guibas, Persistence-Based Clustering in Riemannian Manifolds, 2011

<https://geometrica.saclay.inria.fr/team/Fred.Chazal/papers/cgos-pbc-09/cgos-pbcrm-11.pdf>

Chunyuan Li, Maks Ovsjanikov, Frederic Chazal, Persistence-based Structural Recognition, 2014

<https://geometrica.saclay.inria.fr/team/Fred.Chazal/papers/loc-pbsr-14/CVPR2014.pdf>

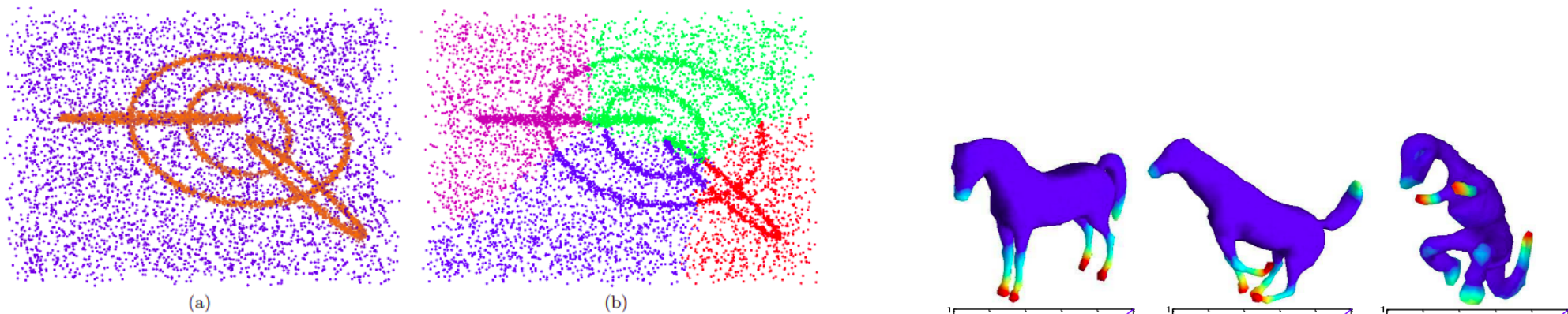
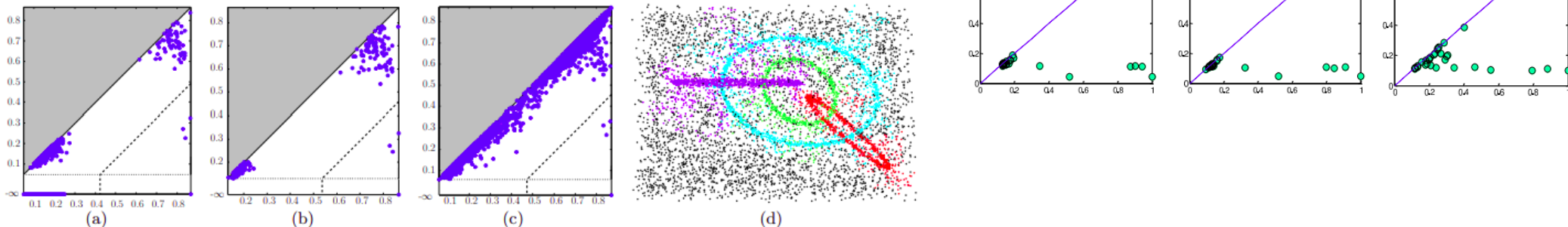


Figure 7: (a) The rings data set with the estimated density function. (b) The result obtained using spectral clustering.



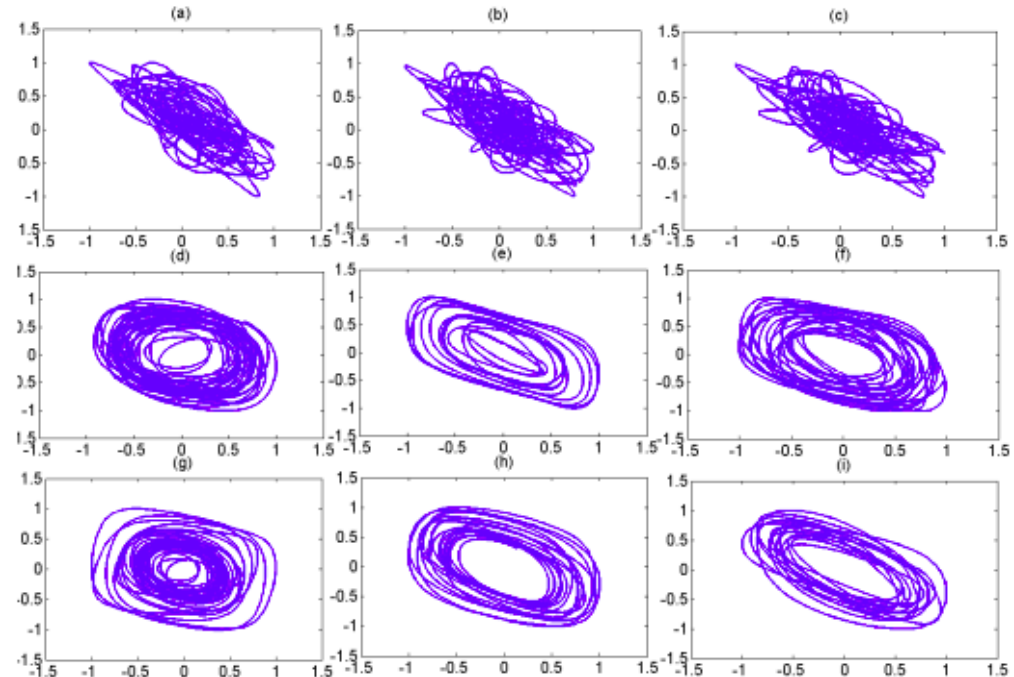
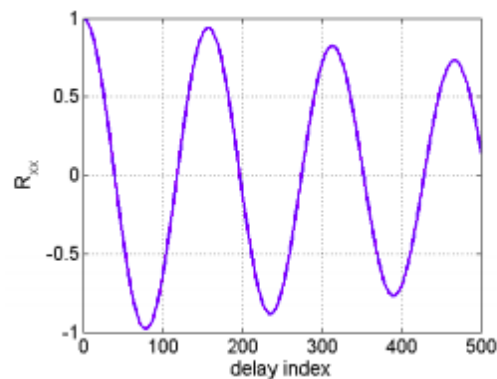
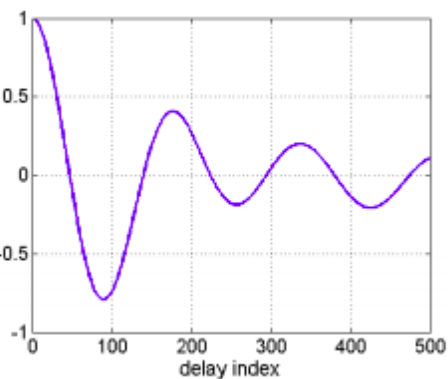
Saba Emrani, Thanos Gentimis, Hamid Krim **Persistent Homology of Delay Embeddings and its Application to Wheeze Detection, 2014**

https://www.researchgate.net/publication/260523931_Persistent_Homology_of_Delay_Embeddings_and_its_Application_to_Wheeze_Detection

→ a time series (x_1, x_2, x_3, \dots) does not contain topology...

turn it into a point cloud of \mathbb{R}^n via **time delay embedding!**

$$X = \{\bar{x}_1, \bar{x}_2, \bar{x}_3, \dots\} \subset \mathbb{R}^n \text{ where } \bar{x}_k = (x_k, x_{k+1}, \dots, x_{k+n-1})$$



I - Variations on persistent homology

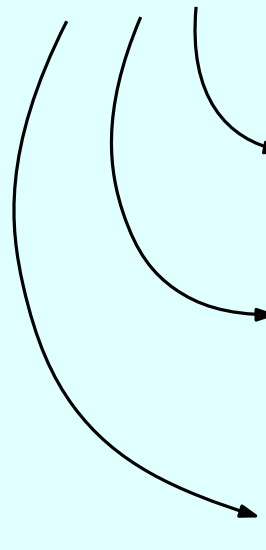
(II - Tutorial)

Download the notebook at

<https://github.com/raphaeltinarrage/EMAp/blob/main/Tutorial3.ipynb>

Conclusion

There are still many things to understand and discover

- 
- algebraic topology tools
 - theory of persistent homology,
foundations of topology inference methods
 - applications in real life



REPÚBLICA FEDERATIVA DO BRASIL

MINISTÉRIO DA EDUCAÇÃO - MEC

UNIVERSIDADE FEDERAL DO PERSISTENT HOMOLOGY

O Reitor da UNIVERSIDADE FEDERAL DO RIO DE JANEIRO, no uso de suas atribuições e tendo em vista a conclusão do Curso de *Engenharia da Computação*, em 21/05/2002, confere o título de *Bacharel em Engenharia da Computação* a

Tu Nombre

cédula de identidade nº. 182910 (órgão expedidor) I.F.P. - RJ

nascido (a) a 13 de Dezembro de 1972 natural Rio de Janeiro

e outorga - lhe o presente Diploma, a fim de que possa gozar de todos os direitos e prerrogativas legais.

Rio de Janeiro, 21 de Maio de 20 02

Reitor

Diretor

Secretário